Exam 1
Chapter 1

Answer the following questions. *You must show your work to receive full credit.* Indicate your final answer with a $\boxed{\text{box}}$.

1. (5 points) Create the truth table for implication; i.e. $p \to q$.

2. Consider the statement "If it is not raining or I have my umbrella, then I am not wet." Let

$$p: \text{"It is raining,"}\quad q: \text{"I have my umbrella," and } r: \text{"I am wet."}$$

(a) (5 points) Write the above statement using symbolic logic.

(b) (5 points) Suppose that r is true; i.e. I am wet. What compound statement involving p and q must be true? (You can use either symbolic logic or you can write out the statement in words.) **Hint:** Use the contrapositive.

3. (10 points) Consider the following statements:

$p =$ "It is snowing today."
$q =$ "It is warmer than yesterday."
$r =$ " We will go skiing."
$s =$ "We will go to the mall.

Prove the following using a two-line proof (a proof table).

$$\left.\begin{array}{c} \neg p \wedge q \\ r \to p \\ \neg r \to s \end{array}\right\} \Rightarrow s$$

Statements	**Reasons**

4. The domain for this problem is some unspecified collection of numbers. Consider the predicate

$$P(x, y) = \text{“}x \text{ is greater than } y\text{.”}$$

(a) (4 points) Translate the following statement into predicate logic.

Every number has a number that is greater than it.

(b) (3 points) Negate your expression from part (a), and simplify it so that no quantifier or connective lies within the scope of a negation.

(c) (3 points) Translate your expression from part (b) into understandable English. Don't use variables in your English translation.

5. Recall that an integer a is even if there exists and integer k such that $a = 2k$. Let n_1 and n_2 be even integers.

(a) (3 points) Write n_1 and n_2 in terms of k_1 and k_2, respectively.

(b) (3 points) Write the product n_1n_2 in terms of k_1 and k_2. Simplify your answer.

(c) (1 point) Is n_1n_2 even or odd?

6. (3 points) What is the difference between an axiom and a theorem?

7. (10 points) Recall that for two integers x and y, we write $x|y$ whenever there exists an integer k such that $y = kx$. Prove the following statement using a direct proof:

Let a, b and c be integers. If $a|b$, then $a|(b \cdot c)$.

8. (10 points) Recall that an integer a is odd if there exists and integer k such that $a = 2k + 1$. Also recall the definition of even on problem 6. Prove the following statement by contraposition:

Let x be an integer. If $x^2 + x + 1$ is even, then x is odd.

9. (10 points) Recall that an angle is obtuse whenever it is greater than $90°$. Use

Theorem 1 *The sum of the measures of the angles of any triangle (in Euclidean geometry) is equal to* $180°$.

to give a proof by contradiction to the following statement:

A triangle cannot have more than one obtuse angle.

Extra Credit 1. (5 points)Recall the axiomatic system of the four-point geometry.

Undefined terms: point, line, is on
Axioms:

1. For every pair of distinct points x and y, there is a unique line l such that x is on l and y is on l.
2. Given a line l and a point x that is not on l, there is a unique line m such that x is on m and no point on l is also on m.
3. There are exactly four points.
4. It is impossible for three points to be on the same line.

Definition 4. A line l **passes through** x if x is on l.

How many distinct lines are there in the four-point geometry? **Hint:** Draw a model if it is helpful.

Extra Credit 2. (5 points) Let a be a logical statement. Is $a \to \neg a$ a contradiction?